HYDRODYNAMIC MODELS AS APPLIED TO THE INVESTIGATION OF MAGNETIC PLASMA STABILITY

E. Ya. Kogan, S. S. Moiseev, and V. N. Oraevskii

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 6, pp. 41–46, 1965

In the present paper magnetohydrodynamic models are employed to investigate the stability of an inhomogeneous magnetic plasma with respect to perturbations in which the electric field may be regarded as a potential field (rot $E \approx 0$). A hydrodynamic model, actually an extension of the well-known Chew-Goldberger-Low model [1], is used to investigate motions transverse to a strong magnetic field in a collisionless plasma. The total viscous stress tensor is given; this includes, together with "magnetic viscosity," the so-called "inertial viscosity."

Ordinary two-fluid hydrodynamics is used in the case of strong collisions $\nu = \omega$. It is shown that the collisional viscosity leads to "flute"-type instability in the case when, collisions being neglected, the "flute" mode is stabilized by a finite Larmor radius. A treatment is also given of the case when epithermal high-frequency oscillations (not leading immediately to anomalous diffusion) cause instability in the low-frequency (drift) oscillations in a manner similar to the "collisional" electron viscosity, leading to anomalous diffusion.

NOTATION

f—particle distribution function; E_α—electric field component; H_0—magnetic field; ρ—density; V—particle velocity; e—charge; m, M—electron and ion mass; Ω_i, Ω_e—ion and electron cyclotron frequencies; $\pi_{\alpha\beta}$—viscous stress tensor; P—pressure; r_i—Larmor radius; $P_{\alpha\beta}$—pressure tensor; t—time; ω—frequency; T—temperature; ν—collision frequency; τ—collision time; j—current density; ω_i, ω_e—ion and electron drift frequencies; k_x, k_y, k_z—wave-vector components; n_0—particle density; g-acceleration due to gravity.

It is well known that the magnetohydrodynamic model for describing a plasma is valid of the particle free path length is much less than the dimension over which the macro-quantities vary, and also that $\nu \gg \omega$, where ν is the collision frequency, ω is the "frequency" of the process ($\omega \sim 1/t_0$, and t_0 is the characteristic time of the process). The hydrodynamic model is suitable for describing motions across the magnetic field in the case where $\nu \ll \omega$ in the presence of strong magnetic fields.

§1. We shall consider how a system of hydrodynamic equations for the motion of a plasma may be obtained when collisions are entirely neglected.

To obtain the system of magnetohydrodynamic equations it is convenient to use Grad's method of moments [2]. The kinetic equation integrated with respect to longitudinal (parallel to the magnetic field) random velocities v_z has the form

$$\frac{\partial f}{\partial t} + v_\alpha \frac{\partial f}{\partial x_\alpha} + \frac{e}{M} E_\beta^* \frac{\partial f}{\partial v_\beta} - \frac{dV_\alpha}{dt} \frac{\partial f}{\partial v_\alpha} + \frac{\partial V_\alpha}{\partial x_\beta} v_\beta \frac{\partial f}{\partial v_\alpha} + \Omega_i [\mathbf{v} \times \mathbf{h}]_\alpha \frac{\partial f}{\partial v_\alpha} = 0,$$

$$\left(E_\beta^* = E_\beta + [\mathbf{V}\times\mathbf{h}]_\beta, \quad \frac{dV_\alpha}{dt} = \frac{\partial V_\alpha}{\partial t} + V_\beta \frac{\partial V_\alpha}{\partial x_\beta}\right). \qquad (1.1)$$

Here M is the ion mass, Ω_i is the ion cyclotron frequency, v is the random part of the particle velocity, V is the mass velocity of the particles (so that $\mathbf{u} = \mathbf{v} + \mathbf{V}$ is the total velocity of the particles); $\mathbf{h}$ is a unit vector in the direction of the magnetic field $\mathbf{H_0}$; the remaining symbols are those in general use; α is an index taking values x, y. By the usual path we obtain from (1.1) a system of equations for moments up to and including the third:

$$\frac{\partial \rho}{\partial t} + \operatorname{div}(\rho \mathbf{V}) = 0, \qquad (1.2)$$

$$\rho\left(\frac{\partial \mathbf{V}_\alpha}{\partial t} + V_\beta \frac{\partial V_\alpha}{\partial x_\beta}\right) = -\frac{\partial p}{\partial x_\alpha} - \frac{\partial \pi_{\alpha\beta}}{\partial x_\beta} + \frac{e}{M}\rho E_\alpha + \Omega_i [\mathbf{V}\times\mathbf{h}]_\alpha, \qquad (1.3)$$

$$\frac{dP_{\alpha\beta}}{dt} + \frac{\partial}{\partial x_\gamma}(S_{\alpha\beta\gamma}) + \frac{dV_\varepsilon}{\partial x_\gamma}(\delta_{\gamma\varepsilon}P_{\alpha\beta} + \delta_{\alpha\varepsilon}P_{\beta\gamma} + \delta_{\beta\varepsilon}P_{\alpha\gamma}) - \Omega_i (P_{\alpha\varepsilon}[e_\varepsilon \times \mathbf{h}]_\beta + P_{\beta\varepsilon}[\mathbf{e}_\varepsilon\times\mathbf{h}]_\alpha) = 0,$$

$$(P_{\alpha\beta} = p\delta_{\alpha\beta} + \pi_{\alpha\beta}), \qquad (1.4)$$

$$\frac{dp}{dt} + 2p \operatorname{div}\mathbf{V} = -\pi_{\alpha\beta}\frac{\partial V_\alpha}{\partial x_\beta} - \frac{1}{2}\operatorname{div} S, \qquad (1.5)$$

$$\frac{dS_{\alpha\beta\gamma}}{dt} + \frac{\partial Q_{\alpha\beta\gamma\varepsilon}}{\partial x_\varepsilon} + \frac{1}{\rho}\frac{\partial P_{\mu\lambda}}{\partial x_\lambda}\{\delta_{\mu\alpha}P_{\beta\gamma} + \delta_{\mu\beta}P_{\alpha\gamma} + \delta_{\mu\gamma}P_{\alpha\beta}\} + \frac{\partial V_\lambda}{\partial x_\mu}(\delta_{\mu\lambda}S_{\alpha\beta\gamma} + \delta_{\alpha\lambda}S_{\mu\beta\gamma} + \delta_{\beta\lambda}S_{\alpha\mu\gamma} + \delta_{\gamma\lambda}S_{\alpha\beta\mu}) + \Omega_i e_{\eta\lambda\mu}h_\lambda\{\delta_{\alpha\mu}S_{\eta\beta\gamma} + \delta_{\beta\mu}S_{\alpha\eta\gamma} + \delta_{\gamma\mu}S_{\alpha\beta\eta}\} = 0,$$

$$\mathbf{e}_1^2 = 1, \quad (\mathbf{e}_1\mathbf{h}) = 0, \quad \mathbf{e}_2 = [\mathbf{e}_1\times\mathbf{h}],$$

$$P_{\alpha\beta} \equiv \int v_\alpha v_\beta f dv, \quad S_{\alpha\beta\gamma} = \int v_\beta v_\beta v_\gamma f dv,$$

$$Q_{\alpha\beta\gamma\varepsilon} \equiv \int v_\alpha v_\beta v_\gamma v_\varepsilon f dv. \qquad (1.6)$$

Here p is the pressure; $P_{\alpha\beta}$ is the ion cyclotron frequency, $\delta_{\alpha\beta}$ is Kronecker's symbol; $\pi_{\alpha\beta}$ is the viscous stress tensor. We note that

$$S_p\pi_{\alpha\beta} \equiv \pi_{xx} + \pi_{yy} = 0, \qquad \pi_{\alpha\beta} = \pi_{\beta\alpha}.$$

Using the equations which have been given it is not difficult to establish that the adiabatic exponent for the given motion $\gamma = 2$ (see also [1]).

For the case $\tau^\circ/t_0 \ll 1$, $r^\circ/L \ll 1$ (τ° is the time of revolution around the Larmor circumference, r° is its radius, L and t_0 are the characteristic spatial and time parameters of the problem), $S_{\alpha\beta\gamma}$ and $Q_{\alpha\beta\gamma\varepsilon}$ may be represented in the form

$$S_{\alpha\beta\gamma} = {}^1/_4(\delta_{\alpha\beta}S_\gamma + \delta_{\alpha\gamma}S_\beta + \delta_{\beta\gamma}S_\alpha),$$

$$Q_{\alpha\beta\gamma\varepsilon} = \frac{pT}{M}(\delta_{\alpha\beta}\delta_{\gamma\varepsilon} + \delta_{\alpha\gamma}\delta_{\beta\varepsilon} + \delta_{\alpha\varepsilon}\delta_{\beta\gamma}) + \frac{T}{M}(\delta_{\alpha\beta}\pi_{\gamma\varepsilon} + \delta_{\alpha\gamma}\pi_{\beta\varepsilon} + \delta_{\alpha\varepsilon}\pi_{\beta\gamma} + \delta_{\beta\gamma}\pi_{\alpha\varepsilon} + \delta_{\beta\varepsilon}\pi_{\alpha\gamma} + \delta_{\gamma\varepsilon}\pi_{\alpha\beta}). \qquad (1.7)$$

Using (1.7), equations (1.4) and (1.6) may be transcribed as follows:

$$\frac{\partial \pi_{\alpha\beta}}{\partial t} + \pi_{\alpha\beta}\operatorname{div}\mathbf{V} - \pi_{\gamma\varepsilon}\frac{\partial V_\gamma}{\partial x_\varepsilon}\delta_{\alpha\beta} + \frac{1}{4}\left(\frac{\partial S_\alpha}{\partial x_\beta} + \frac{\partial S_\beta}{\partial x_\alpha} - \delta_{\alpha\beta}\operatorname{div}\mathbf{S}\right) +$$

$$+ \pi_{\alpha\mu}\frac{\partial V_\beta}{\partial x_\mu} + \pi_{\beta\mu}\frac{\partial V_\alpha}{\partial x_\mu} + p\left(\frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial V_\beta}{\partial x_\alpha} - \delta_{\alpha\beta}\operatorname{div}\mathbf{V}\right) -$$

$$- \Omega_i\{\pi_{\alpha\mu}[\mathbf{e}_\mu\times\mathbf{h}]_\beta + \pi_{\beta\mu}[\mathbf{e}_\mu\times\mathbf{h}]_\alpha\} = 0,$$

$$\frac{dS_\alpha}{dt} + 4\frac{P_{\alpha\beta}}{M}\frac{\partial T}{\partial x_\beta} + 2p\frac{\partial}{\partial x_\beta}\left(\frac{\pi_{\alpha\beta}}{\rho}\right) + \frac{3}{2}\left\{S_\alpha\operatorname{div}\mathbf{V} + S_\beta\frac{\partial V_\alpha}{\partial x_\beta} + \frac{1}{3}S_\beta\frac{\partial V_\beta}{\partial x_\alpha}\right\} - \Omega_i[S\times\mathbf{h}]_\alpha = 0. \qquad (1.8)$$

Retaining linear terms in (1.8) for perturbations of quantities of the form $\exp(i\omega t)$, we obtain an expression for the viscous stress tensor by means of an expansion in $\omega/\Omega_i \ll 1$:

$$\pi_{yy} = -\pi_{xx} = \frac{p}{2\Omega_i}\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right) + \frac{i\omega p}{4\Omega_i^2}\left(\frac{\partial V_x}{\partial x} - \frac{\partial V_y}{\partial y}\right) - \frac{1}{2\Omega_i^2}\left\{\frac{\partial}{\partial y}\left(\frac{p}{M}\frac{\partial}{\partial y}\right) - \frac{\partial}{\partial x}\left(\frac{p}{M}\frac{\partial}{\partial x}\right)\right\}T, \qquad (1.9)$$

$$\pi_{xy} = \pi_{yx} = \frac{p}{2\Omega_i}\left(\frac{\partial V_x}{\partial x} - \frac{\partial V_y}{\partial y}\right) - \frac{i\omega p}{4\Omega_i^2}\left(\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x}\right) - \frac{1}{2\Omega_i^2}\left\{\frac{\partial}{\partial x}\left(\frac{p}{M}\frac{\partial}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{p}{M}\frac{\partial}{\partial x}\right)\right\}T,$$

$$S_x = -\frac{1}{\Omega_i}\left\{\frac{4p}{M}\frac{\partial T}{\partial y} + 2p\frac{\partial}{\partial x}\left(\frac{\pi_{xy}}{\rho}\right) + 2p\frac{\partial}{\partial y}\left(\frac{\pi_{yy}}{\rho}\right)\right\},$$

$$S_y = \frac{1}{\Omega_i}\left\{\frac{4p}{M}\frac{\partial T}{\partial x} + 2p\frac{\partial}{\partial x}\left(\frac{\pi_{xx}}{\rho}\right) + 2p\frac{\partial}{\partial y}\left(\frac{\pi_{xy}}{\rho}\right)\right\}. \qquad (1.10)$$

Here T is the temperature, ρ the density.

In (1.9) terms of the first order in ω/Ω_i correspond to the expressions for the "magnetic" viscosity obtained in [3].

The following term describes the so called "inertial" viscosity. We note that the expression for the viscous stress tensor in [3] does not have a limiting transition to the case $\nu_i \to 0$ (ν_i is the ion-ion collision frequency). For use in what follows we shall write out these expressions in the "collisionless" case for $\Omega_{i\tau} \gg 1$ in a coordinate system with its z axis parallel to the magnetic field H_0 [3]

$$\pi_{xx} = -\frac{p}{2\nu_i}(W_{xx} + W_{yy}) - \frac{0.3p\nu_i}{2\Omega_i^2}(W_{xx} - W_{yy}) - \frac{p}{2\Omega_i}W_{xy}, \quad \pi_{yy} = -\frac{p}{2\nu_i}(W_{xx} + W_{yy}) - \frac{0.3p\nu_i}{2\Omega_i^2}(W_{yy} - W_{xx}) + \frac{p}{2\Omega_i}W_{xy},$$

$$\pi_{xy} = \pi_{yx} = -\frac{0.3p\nu_i}{\Omega_i^2}W_{xy} + \frac{p}{2\Omega_i}(W_{xx} - W_{yy}),$$

$$W_{\alpha\beta} = \frac{\partial V_\alpha}{\partial x_\beta} + \frac{\partial V_\beta}{\partial x_\alpha} - \frac{2}{3}\delta_{\alpha\beta}\operatorname{div}\mathbf{V}. \qquad (1.11)$$

§2. We shall investigate the "flute" instability in two limiting cases: for collisonless plasmas and for a high collision frequency. In the first case, using equations (1.2), (1.3), (1.5), (1.9), (1.10), taking into account the gravitational potential, the quasi-neutrality of the plasma and the potentiality of the respective disturbances

$$\operatorname{div}\mathbf{j} = 0, \qquad \operatorname{rot}\mathbf{E} = 0, \qquad (2.1)$$

we may obtain, retaining terms up to $(\omega/\Omega_i)^2$ inclusive, the following expression for perturbations of the type $\varphi(x)\exp(i\omega t + ikr)$ (the plasma is assumed to be inhomogeneous in the x direction);

$$\frac{3}{4}\frac{\omega}{\omega-\omega_i}\frac{\partial^4\varphi}{\partial x^4} - k_y^2\left(\frac{3}{2}\frac{\omega}{\omega-\omega_i} - \frac{gk_0}{(\omega-\omega_i)^2} - \alpha\frac{\omega}{\omega-\omega_i}\right)\frac{\partial^2\varphi}{\partial x^2} + k_y^4\left\{\frac{3}{4}\frac{\omega}{\omega-\omega_i} - \frac{gk_0}{(\omega-\omega_i)^2} - \alpha\left[\frac{\omega}{\omega-\omega_i} - \frac{gk_0}{(\omega-\omega_i)^2}\right]\right\}\varphi = 0,$$

$$\omega_i = k_y k_0\frac{cT}{eH}, \qquad k_0 \equiv \frac{h_0'}{n_0}, \qquad \alpha = \frac{1}{k_y^2 r_i^{\circ 2}}. \qquad (2.2)$$

Equation (2.2) is written in the laboratory frame of reference with the z axis parallel to the magnetic field. Here g is the acceleration due to gravity, ω_i is the ion drift frequency, n_0 is the unperturbed density, r° is the ion Larmor radius, and the prime dash denotes differentiation with respect to x.

We shall investigate the solutions of (2.2) on the assumption that $\omega_i^2 > gn_0'/n_0$, i.e., when solutions of the second-order differential equation do not lead to instability. The presence of zeros of U_2 (U_2 is the coefficient of the second derivative), may lead, as is well known [4], to the complication of all four solutions and to the appearance of new dispersion properties of the plasma. For sufficiently short waves ($\lambda_2 \gtrsim r^\circ$) it is clear from (2.2), that we shall obtain a qualitatively valid result by neglecting the $\sim\varphi$ term and analyzing the solutions of $U_2 = 0$. Here we see that as distinct from [4] unstable solutions are absent for two-dimensional motion in a purely "collisionless" situation. We note that if $\omega_i^2 \lesssim gk_0$, i.e., the increments of the unstable solution are small, then the points $U_2 \approx 0$ lie in the neighborhood of the real axis. In this case the solutions become complicated and the integral contribution due to the $k_2 \sim \sqrt{\alpha U_2}$ mode for finite solutions may turn out to be the most important, which leads to a better stability criterion

$$\omega_i^2 > gk_0/\alpha.$$

In order to obtain an equation in the case of strong collisions we make use of relations (1.2), (1.3), (1.11) in a gravitational field. For perturbations like $\varphi(x)\exp(i\omega t + ik_y y)$ we obtain for $\Omega_{i\tau} \gg 1$

$$\frac{\partial^4\varphi}{\partial x^4} + \frac{1}{r_i^2}\left(\frac{\omega-\omega_i}{i\nu_i} - 2\beta\right)\frac{\partial^2\varphi}{\partial x^2} + \frac{k_y^2}{r_i^2}\varphi\left(-\frac{\omega-\omega_i}{i\nu_i} + \frac{gk_0}{i\nu_i\omega} + \beta\right) = 0,$$

$$\beta = 1/\alpha = k_y^2 r^{\circ 2}_i. \qquad (2.3)$$

Here ν_i is the ion-ion collision frequency.

When the effect of the fourth derivative is insignificant, we obtain a qualitatively correct result for finite solutions from the condition $U_1 \approx 0$, as is well known [4] (U_1 is the coefficient of φ), giving

$$\omega^2 - \omega(\omega_t + i\beta\nu_i) - gk_0 = 0. \qquad (2.4)$$

We see that (2.4) also contains unstable solutions even in the case $\omega_i^2 > gn_0'/n_0$ when flute instabilities are stabilized in the absence of collisions. Moreover, when $\omega_i^2 \gg gk_0$ we have

$$\mathrm{Re}\,\omega \approx -\frac{gk_0}{\omega_i}, \qquad \mathrm{Im}\,\omega \sim -\frac{|gk_0|\nu_i\beta}{\omega_i^2}. \qquad (2.5)$$

It is important to note that equation (2.3) will contain unstable solutions if another instability, instead of the usual collisions, leads to viscous dissipation, which will lead formally to at least another expression for the "viscosity" coefficient. (As is now well known, A. V. Timofeev and D. I. Ryutov have also called attention to the effect of collective viscosity in the case of flute instabilities.) The presence of zeros of U_2 makes it necessary to take the fourth derivative into account, which lessens the increment of the given instability, as can easily be seen.

§3. It has been partially demonstrated in the foregoing section that taking ion-ion collisions into account may lead to the development of flute instabilities even under conditions when the ion Larmor radius is large ($\omega_i^2 > gk_0$) It is well known that taking dissipative effects into account in a series of other cases (see, for example, [5–7]) may lead to the appearance of new instabilities. Thus electron-ion friction leads to the development of the drift-dissipation instability, giving rise to Bohm [5] diffusion. All these instabilities due to taking collisions into account are not very dangerous in a high-temperature system when the collision frequency falls. It must be kept in mind, however, that dissipative effects may arise on account of certain other "seeding" instabilities, and then, even in the absence of the usual collisions, generally speaking conditions are created for the propagation of dissipative instabilities. The possibility of such an effect has previously been noted by other authors.*

Her we shall consider the effect of high-frequency oscillations on the development of the drift-dissipative instability.

First of all we draw attention to the fact that the "longitudinal" electron viscosity leads to a destabilization of the plasma in drift waves similar to electron-ion friction. Actually,

$$-ik_z n_0 T_0 - en_{0e}E_z + \eta\Delta_{11}v - mn_{0e}v_{0z}\nu_{ei} = 0 \quad (3.1)$$

from the equation for longitudinal electron motion, where k_z is the wave vector along z, e, the charge, n_{0e} the unperturbed electron density, and ν_{ei} the effective electron-ion collision frequency. It is clear that the term $\eta\Delta_{11}v$ is analogous to the term characterizing ion-electron friction, and consequently exerts a destabilizing effect. However, under the conditions of applicability of ordinary hydrodynamics the viscous term in $(\lambda_e/\lambda_{11})^2$ is less than the electron-ion friction on account of pair collisions, and is thus insignificant. Let us now consider collisions to be absent, and let high-frequency electron oscillations, caused by beam instability, be present and lead to a certain effective "viscosity" of the electron gas (interaction with such oscillations is known to be equivalent to electron-electron collisions [8]). By way of example, we shall consider the beam instability investigated in [9]. The quasilinear equation for the beam has the form

$$v\frac{\partial f}{\partial x} = \frac{e^2}{m^2}\frac{\partial}{\partial v}\left(\frac{|E_k|^2}{v}\frac{\partial f}{\partial v}\right), \qquad (3.2)$$

where f is the particle distribution function for the beam, $|E_k|$ is the amplitude of field pulsations, according to [9] of order of magnitude

$$|E_k| \sim n_0 m\frac{v^3}{\omega_0}\exp\left(\frac{t}{\tau_p}\right) \qquad \left(\omega_0 = \left(\frac{4\pi N_{0e}e^2}{m}\right)^{1/2}\right). \quad (3.3)$$

Here n_0 is the unperturbed beam density, ω_0 is the plasma oscillation frequency, N_{0e} is the unperturbed plasma electron density, τ_p is the time for establishing a "plateau" in the function f.

Nonlinear wave interactions are still insignificant in the case where $n_0 \ll N_{0e}$, and we may employ a system of hydrodynamical equations for the plasma electrons (see [9]).

In order to evaluate the influence of high-frequency oscillations on the drift-dissipative instability, we must in fact calculate the effective τ (time) of electron collisions in the beam. From (3.2) and (3.3) we obtain

$$\tau \sim \frac{\sqrt{N_{0e}}}{n_0}\frac{1}{\omega_0^*} \qquad (3.4)$$

for $\tau \sim \tau_p$.

Here ω_0^* is the plasma frequency for the beam density. Expression (3.4) is valid when the resonance electrons have velocities of the order of the mean thermal velocities.

Further, for cases of low-frequency potential perturbations $\exp(i\omega t + ik_y y + ik_z z)$, we make use of the following relations.

The equation of motion of cold ions across the magnetic field

$$MN_{0i}\frac{\partial \mathbf{v}_i}{\partial t} = eN_{0i}\mathbf{E}_\perp + \frac{e}{c}N_{0i}[\mathbf{v}_i \cdot \mathbf{H}_0]. \qquad (3.5)$$

The equation of motion of the beam electrons

$$-ik_z nT_0 - en_0E_z - k_z^2\tau n_0 T_0 v = 0.$$

The condition of quasi-neutrality

$$\mathrm{div}\,\mathbf{j} = 0. \qquad (3.7)$$

We may then obtain the dispersion equation (motion) of the plasma electrons along $\mathbf{H}_0$:

* G. M. Zaslavskii, S. S. Moiseev, and R. A. Sagdeev, Second All-Union Congress of Mechanics, Moscow, January 1964.

$$1 - i\frac{\omega_s}{\omega k_z^2 \lambda_e^2}\frac{n_0}{N_{0e}}\left[1 - \frac{\omega_e}{\omega}\right] = 0,$$

$$\omega_s = \frac{\Omega_i \Omega_e}{\nu_e}\frac{k_z^2}{k_y^2}, \quad \omega_e = \frac{N_{0e}'}{N_{0e}} k_y \frac{cT}{eH}. \qquad (3.8)$$

Here n, N_e, N_i are the perturbed density of beam electrons, plasma electron and ions, respectively, T is the temperature, ν_e is the electron cyclotron frequency, ν_e is the "electron" collision frequency, λ_e is the electron free path ($\lambda_e \sim v\tau$, v is the thermal velocity of electrons), ω_e is the electron drift frequency, N_{0e}' is the derivative of N_{0e} with respect to x (the direction of the inhomogeneity).

The condition that the Landau damping contribution for electrons be small is of the form

$$\frac{\omega^2}{k_z^2 v^2} \ll \frac{\omega_s}{\omega k_z^2 \lambda_e^2}\frac{n_0}{N_{0e}}. \qquad (3.9)$$

Then for $\omega_s^* \gg \omega_e$, $\omega_0^*(n_0 / N_{0e})^{-1/2} \gg \omega_e$ (the last inequality is the condition of applicability of hydrodynamics to the drift motions of the beam electrons), we obtain

$$\mathrm{Im}\,\omega \sim \frac{\omega_e^2 k_z^2 \lambda_e^2 N_{0e}}{\omega_s n_0}, \qquad \omega_s^* = \frac{\omega_s n_0}{k_z^2 \lambda_e^2 N_{0e}}. \qquad (3.10)$$

We note that the increment does not depend on k_z, and so the crossing of lines of force of the magnetic field does not exert an immediate influence on the development of instability within the limits of applicability of the given result.

Thus high-frequency oscillations which do not lead directly to diffusion may still act as an indirect cause of diffusion at low-frequency oscillations.

The authors are grateful to A. A. Galeev for valuable discussion.

REFERENCES

1. G. Chew, M. Goldberger, and F. Low, "The Boltzmann equation and one-fluid hydromagnetic equations in the absence of particle collisions," Proc. Roy. Soc., vol. 236, p. 112, 1956.

2. S. Chapman and T. Cowling, The Mathematical Theory of Nonuniform Gases [Russian translation], Izd. inostr. lit., 1960.

3. S. I. Grabinskii, "Transport phenomena in plasma," collection Problems of Plasma Physics [in Russian], vol. 1, 1963.

4. G. M. Zaslavskii, S. S. Moiseev, and R. Z. Sagdeev, "Asymptotic methods of hydrodynamic stability theory," PMTF, no. 6, 1964.

5. S. S. Moiseev and R. Z. Sagdeev, "Bohm's diffusion coefficient," ZhETF, vol. 44, no. 2, p. 763, 1963.

6. A. A. Galeev, V. N. Oraevskii, and R. Z. Sagdeev, "Universal plasma instability," ZhETF, vol. 44, no. 3, p. 903, 1963.

7. A. A. Galeev, S. S. Moiseev, and R. Z. Sagdeev, "Theory of stability of an inhomogeneous plasma and anomalous diffusion," Atomnaya energiya, vol. 6, 1963.

8. B. B. Kadomtsev, collection Problems of Plasma Theory [in Russian], Moscow, vol. 4, 1964.

9. Ya. B. Fainberg and V. D. Shapiro, "Quasilinear theory of excitation of oscillations on injection of an electron beam into a plasma half-space," ZhETF, vol. 47, no. 4 (10), p. 1388, 1964.

28 June 1965 Novosibirsk